Funded and unfunded systems: two ends of the same stick

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SUMMARY

The aim of this article is to give evidence that, if a Pension Fund of the defined contribution type is brought back to a “sustainable level” of liability uncovered, as regard to the contribution level, and yields yearly a revenue on the pension savings equalling the asset’s interest rate plus wage’s growth rate respectively weighing as its funded and unfunded liability components on the total pension debt, then the Fund can guarantee sustainability through time.

Key words: pension system
pay-as-you go
funded system,
social insurance

1. INTRODUCTION

In 1966 Henry Aaron, following an indication by Prof. Samuelson, proved the "paradox of social security" by which "social insurance can increase the welfare of each person if the sum of the rates of growth of population and real wage exceeds the rate of interest."

Since then the trust in the pay-as-you go pension system has largely grown, until a fall in the demographic level took place in almost all western countries, proving the unsustainability, in the medium long term, of such system at the current benefit levels.

The answer to the demographic problem has been the introduction of new funded systems through a second and third pillar, permitting the first pillar to progressively reduce its benefits which would be reintegrated by the new two-pillar benefits.

The aim of the present article is to prove that in a defined contribution Pension Fund, of pay-as-you-go type, negative demographic effects can be opposed through the introduction in the same system of a funded component, and that it is possible to run pension obligations through a sort of mixed financial management capable of securing sustainability and more adequate benefits.

In this way also the older worker generations, who will have a good level of benefits, if compared to contribution amounts, made possible by a favourable demographic factor, will participate in refunding.

The new scheme will favour intergenerational equity in the contribution to benefit ratio, thus fulfilling the task of solidarity on which the pay as you go pension system rests. The funded component, that we can define *a differential reserve*, must guarantee the system's sustainability with an acceptable contribution level and, besides, providing an interest rate, may guarantee more adequate pension saving returns even if the population growth rate falls.

This article highlights the fact that both types, the funded and pay-as-you-go financial managements, are not to be seen antithetical, as it commonly happens, but working as the "two ends of the same stick", in a "continuum".

It points out, moreover, that the prevailing of either the one or the other scheme must follow a more favourable expected development in wages or in asset interest rate.

The approach to the problem consists in considering the system's total liability (composed of its latent part towards workers and its current part towards pensioners) as the result of two components, one 'covered' by the asset, therefore funded, and the remaining part 'uncovered' therefore 'unfunded'.

In this respect two main hypotheses are formulated.

The former concerns the liability's unfunded component which, respect to wages, must be consistent with the amount of the contribution level. In that sense we speak of a *"sustainable level of unfunded pension liability"*.

The latter concerns the revenue on the pension savings.

We assume that the revenue in question equals each year the convex combination of the asset's interest rate and wage's growth rate which weigh as the funded and unfunded components of total liability.

We prove that, if the system is brought back to a sustainable level of unfunded pension liability by introducing an adequate funded component and the revenue on the pensions savings is consistent with the typology of the pension liability in the sense specified, then its asset will never be zeroed and hence the system will be solvent through time.

In Italy a funded component, substantial but not sufficient, is already present in the liberal professions social security systems: the target is to make it structural also by transforming the systems from defined benefits to defined contributions.

2.SUSTAINABILITY THEOREM

All the functions here contained are supposed to be continuous and, when necessary differentiable in $\left[t^*,+\infty\right)$,"control" time interval of the theorem.

Let's consider the asset's $F(t)$ evolution course resulting from its interest rate at time t, from contributions (which we consider already free from the Fund's management costs) and from pension obligations. We indicate by:

$r(t)$ = The asset instant interest rate at time t, (on a yearly basis).

$C(t)$ = Instant flow of contributions at time t, (on a yearly basis)

$P(t)$ = Instant flow of pensions at time t, (on a yearly basis)

We assume:

$$\dot{F}(t)=F(t)r(t)+C(t)-P(t) \qquad (1)$$

Let's write the evolution equation of the Pension Fund's total liability $L^T(t)$.

We have:

$$L^T(t) = L^a(t) + L^P(t)$$

where

$L^a(t)$ = Pension liability at time t towards workers (in activity), *latent debt.*

$L^P(t)$ = Pension liability at time t towards pensioners, *current debt.*

We indicate:

${}^aL^P(t)$ = Instant flow of pension liability (on a yearly basis) at time t which turns from latent into current.

$r_L(t)$ = Instant interest credited by the Fund at time t on pension liability (both current and latent) on a yearly basis.

We observe that $r_L(t)$ indicates, as to workers, the revaluation of total pension savings while, as to pensioners, it indicates the revaluation rate of their pensions. Let's suppose, in fact, that the initial pension be computed in contribution terms on the basis of actuarial equilibrium between contributions and pensions at a pre-paid interest rate, *technical rate*, equal to zero. The initial pension therefore results from total pension savings at retirement time divided by the annuitization divisor at zero technical rate which therefore, in case a survivor pension is not paid out, coincides with his remaining average life expectancy at that time.

Let's write the evolution equation of the pension debt component relative to workers:

$$\dot{L}^a(t) = L^a(t)r_L(t) + C(t) - {}^aL^P(t)$$

In a parallel way, as to current debt towards pensioners, it issues:

$$\dot{L}^P(t) = L^P(t)r_L(t) + {}^aL^P(t) - P(t)$$

As to total debt, we have the following evolution equation:

$$\dot{L}^T(t) = L^T(t)r_L(t) + C(t) - P(t) \qquad (2)$$

The above formula (2) provides the evolution of the Fund's total pension liability, apart from accidental deviations, therefore un-systematic, in a contribution type pension scheme based on an actuarial contribution - pension equilibrium, provided that demographic evaluations are correct.

We point out that, in our assumptions, *the whole contribution amount* turns into Fund pension liability, that is into pension benefits for subscribers, including, for example, also the contributions of those who died in the course of their working years.

We indicate by $\nu(t)$ the quotient at time t between total pension liability and current debit, i.c.

$\nu(t) = \dfrac{L^T(t)}{L^P(t)}$ hence $L^P(t) = \dfrac{L^T(t)}{\nu(t)}$; it issues $\nu(t) \geq 1$

If we furthermore suppose, in order to simplify the matter without losing the overall view, that pension benefits are not extended to survivors[1], we take:

$\gamma(t)$ = weighed average residual life expectancy at time t relative to the pensioners population. The weigh relative to each pensioner equals that of his pension benefit on that of current pensions whole number.

It issues:

$P(t) = \gamma^{-1}(t)L^P(t)$ and therefore

$$P(t) = \left[(\gamma(t)\nu(t))^{-1} \cdot L^T(t)\right]$$

We define the Funds's 'unfunded' pension liability its uncovered part, that is:

$$L^{PAYG}(t) = L^T(t) - F(t) \qquad (F(t) \leq L^T(t))$$

If we suppose that the population of the insured is made up of workers with a wages of their own, let's put:

$\beta(t) = \dfrac{L^{PAYG}(t)}{W(t)}$ where

$W(t)$ = Instant flow of wages at time t (yearly basis), $W(t) > 0$, for $t \geq t^*$.

Let's furthermore define, as to contributions

$\alpha(t)$ = contributions rate at time t $\qquad (\alpha(t) > 0)$

Taking into account the hypothesis $W(t) > 0, \ \text{for } t \geq t^*$, we prove the following:

Theorem

Let's suppose that at time t^* results $F(t^*) > 0$ and that the *unfunded liability proves sustainable*, as regard to the contribution level, in other terms it satisfies the condition:

a) $$\frac{\beta(t^*)}{\gamma(t)\nu(t)} \leq \alpha(t^*) \qquad \text{for } t \geq t^*:$$

The following condition holds:

b) The Fund pays a pension liability with an interest rate equalling:

$$r_L(t) = r(t)\frac{F(t)}{L^T(t)} + \frac{\dot{W}(t)}{W(t)} \cdot \frac{L^T(t) - F(t)}{L^T(t)} \qquad \text{for} \qquad t \geq t^*$$

Then, if starting from time t^* a contribution rate $\alpha(t) = \alpha(t^*)$ is applied, the asset can't be zeroed, i.e.

$$F(t) > 0 \qquad \forall\, t, t > t^*$$

Evidence:

Let's first verify that, if starting from time t^* the instant interest rate $r_L(t)$ paid by the Fund on pension liability is provided by (b), then it follows:

$$\beta(t) = \beta(t^*) \qquad \text{where} \quad t > t^*$$

Since

$$\beta(t) = \frac{L^{PAYG}(t)}{W(t)},$$

We verify that, for $t \geq t^*$

$$\dot{\beta}(t) = \left(\frac{\dot{L}^{PAYG}(t)}{W(t)} \right) = 0$$

We have

$\dot{L}^{PAYG}(t) = \dot{L}^{T}(t) - \dot{F}(t) =$ (as a result of (2) and (1))

$= L^T(t) r_L(t) + C(t) - P(t) - F(t) r(t) - C(t) + P(t) =$

$= L^T(t) r_L(t) - F(t) r(t)$

We observe that, from (b), it also issues:

$$L^T(t) \cdot r_L(t) - F(t) r(t) = \left(L^T(t) - F(t)\right) \cdot \frac{\dot{W}(t)}{W(t)} \qquad \text{for} \quad t \geq t^*$$

As a result of what indicated above, it follows:

$$\dot{L}^{PAYG}(t) W(t) - L^{PAYG}(t) \cdot \dot{W}(t) = 0 \qquad \text{for} \quad t \geq t^*$$

and therefore

$$\left(\frac{\dot{L}^{PAYG}(t)}{W(t)}\right) = 0, \text{ for } t \geq t^*$$

and hence $\beta(t) = \beta(t^*)$ for $t > t^*$.

We reformulate (1) expressing contributions by means of wage and contribution rate, that is $C(t) = \alpha(t) W(t)$ and, besides, writing pension benefits starting from the total pension liability, we have:

$$\dot{F}(t) = F(t) r(t) + C(t) - P(t)$$

$$\dot{F}(t) = F(t) r(t) + \alpha(t) W(t) - \gamma^{-1}(t) L^P(t) =$$

$$= F(t) r(t) + \left[\alpha(t) W(t) - (\gamma(t) \nu(t))^{-1} \cdot L^T(t)\right] =$$

$$= F(t) r(t) + \left[\alpha(t) W(t) - (\gamma(t) \nu(t))^{-1} \left(L^T(t) - F(t)\right)\right] - (\gamma(t) \nu(t))^{-1} F(t) =$$

$$= F(t) r(t) + \left[\alpha(t) W(t) - (\gamma(t) \nu(t))^{-1} L^{PAYG}(t)\right] - (\gamma(t) \nu(t))^{-1} F(t)$$

Replacing $L^{PAYG}(t)$ with $\beta(t) W(t)$, we have:

$$\dot{F}(t) = F(t)\left[r(t) - (\gamma(t) \nu(t))^{-1}\right] + W(t)\left(\alpha(t) - \frac{\beta(t)}{\gamma(t) \nu(t)}\right) \qquad \text{(1 bis)}$$

Considering that from (b), follows $\beta(t)=\beta(t^*)$ for $t>t^*$, and that, starting from time t^*, the contribution rate $\alpha(t)=\alpha(t^*)$ is applied, it issues that:

$$\dot{F}(t)=F(t)\left(r(t)-\frac{1}{\gamma(t)\nu(t)}\right)+W(t)\left(\alpha(t^*)-\frac{\beta(t^*)}{\gamma(t)\nu(t)}\right) \qquad \text{for } t\geq t^*$$

With initial condition, for $t = t^*$, we obtain:

$$F(t)=e^{\int_{t^*}^{t}\left(r(\tau)-\frac{1}{\gamma(\tau)\nu(\tau)}\right)d\tau}\cdot\left(F(t^*)+\int_{t^*}^{t}e^{-\int_{t^*}^{\tau}\left(r(s)-\frac{1}{\gamma(s)\nu(s)}\right)ds}W(\tau)\left(\alpha(t^*)-\frac{\beta(t^*)}{\gamma(\tau)\nu(\tau)}\right)d\tau\right) \quad (3)$$

for $t\geq t^*$

Taking into account hypothesis a), formula (3) implies that the asset $F(t)$, active in t^*, can't be zeroed.

We can say the asset $F(t^*)$ is a differential reserve, respect to the contribution level $\alpha(t^*)$, for the Fund at time t^* if hypothesis a) is verified.

OBSERVATION

In theorem, we can apply a non constant contribution rate α (t) that verifies this condition

$$\frac{\beta(t^*)}{\gamma(t)\nu(t)}\leq\alpha(t) \qquad \text{for } t\geq t^* \quad .$$

In this way, we can eliminate the hypothesis a) and however the theorem thesis works.

3. CONCLUSIONS

In this paper we prove that it is possible to ensure the equilibrium of defined contribution pension systems with a component of pension debt unfunded if, with respect to wages, the unfunded component is sustainable as regard to the contribution level and the systems provide an interest rate on pensions savings coherent with the funded and unfunded debt components.

[1] When also a survivor pension is paid out in addition to a primary pension, $\gamma(t)$ equals the average weighed "remaining life expectancy" of all pensions in course at time t, taking into account, as regards primary pensions, also the expected duration of the correlated survivor benefit and its possible inferior amount with respect to the primary pension benefit. The weight relative to each pension equals that of his pension benefit on that of current pensions whole number.

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